credited to Guido Arezzo by Wailly[1] on the authority of the Benedictines, and also in the Nouveau traitè Diplomatique, in which the work is given as a computation on the sand table. This is the more likely inasmuch as several Arabic works written before 980[2] refer to calculation on a table, and to calculation without erasure. There is also a Computus of the XI century by Marianus in the Trinity College Library, bound with some arithmetic of the XII century.[3]

REVIEW OF A SIGNIFICANT TEXT IN CALCULUS.*

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This little book, entitled Elements of Differential and Integral Calculus, deserves more than a passing notice, especially because it is intended for a class of students differing considerably from those who now study elementary calculus in our colleges and universities. As stated in the preface, it is the result of a series of lectures delivered during the last six years to classes consisting chiefly of students of engineering and chemistry. In working over the lectures "it soon appeared," continues the preface, "that the mathematical knowledge which needs to be possesseed by a student before attempting the calculus is very much less than has been supposed. For example, the binomial theorem of algebra and the addition theorem of trigonometry are quite unnecessary. This book is written with the view of making the subject more easily and generally accessible than it has been hitherto. The principles of the differential and integral calculus ought to be counted as a part of the intellectual heritage of any educated man or woman in the twentieth century no less than the Copernican system or the Darwinian theory. In order to make a beginning no previous knowledge of mathematics is needed beyond the most elementary notions of geometry, a little algebra, including the law of indices and the definitions of the trigonometric funetions."

The general mode of presentation bears striking evidence of the influence of recent discussions of mathematical pedagogy. The consideration of each new process is preceded by concrete and simple problems in the solution of which the process is required. The simplicity of the problems themselves leaves the mind free to concentrate upon the one new thing offered for consideration, namely, the process of solution.

1). Elements de Paleographie, Paris, 1838, vol. I, pp. 711–716.

2). *M. Suter, Abhandlungen zur Geschichte der Mathematik, Fihrist,* pp. 37, 40, 41.

3). *Trinity College Library, Catalog of Manuscripts.*

*Elements of the Differential and Integral Calculus by A. E. H. Love. Cambridge University Press, pages 204+XIII.

The subject of coordinate geometry is introduced by such problems as the representation of the relation between the length of a spring and the weight of a body suspended from it, the relation between the readings of Centigrade and Fahrenheit thermometers, and the distances covered by a falling body in different intervals of time. The notion of "gradient" or "slope" is emphasized first in connection with the straight line and then in connection with the parabola obtained from the falling body problem. Numerous little exercises are interspersed for finding the gradient of straight lines and simple curves.

Chapter II deals with differentiation more formally under the following heads:

(a) Falling bodies.
(b) Speed of advancing body.
(c) Rates of change in general.
(d) Tangents to a curve.

Up to this point not a word has been said as to what is meant by "limit," though the word and the idea have been freely used. We now read (p. 21): "We shall be able to proceed more quickly afterward, and we shall be more certain that our work is correct, if we take a little time to think exactly what it is that we mean when we say that a function of h tends to a limit as h tends to zero." Then follows a beautifully simple exposition. Thus at every point the student is taken fully into the confidence of the author.

After the proofs of the formula for differentiating u^n for all rational values of n the second derivative is introduced and this is followed immediately by applications to tangents, approximations such as the use of $x^n+nx^{n-1}h$ for $(x+h)^n$, and more generally the use of $f(x)+hf'(x)$ for $f(x+h)$, maxima and minima including the use of $f''(x)$ to discriminate between a maximum and a minimum, and the mean value theorems.

Chapter IV (p. 55) first introduces integration. Practically throughout the book integration is regarded as the inverse process of differentiation and not as finding the limit of a sum. To find an area it is first asked "what is the derivative of the area?" and similarly for finding volumes, surfaces, lengths of areas.

Chapter VI, which deals with logarithms and exponential functions, is more complicated and difficult than the rest of the book, as are also the problems and applications of this chapter.

Chapter VII consists of a treatment of trigonometric functions. The proofs are unique inasmuch as little or no use is made of trigonometric formulas beyond the mere definitions of the functions. The theorems of this chapter are immediately applied to problems in oscillatory motion.

Chapter VIII deals with methods of integration and Chapter IX with applications to length of curves, curvature, and areas of surfaces of revolution.

In Chapter X (p. 157) the definite integral is considered as a limit of a sum, but not even here is it defined as such a limit. It is simply shown that the definite integral as previously defined is indeed the limit of a certain sum.

Chapter XI deals with centers of gravity, centers of pressure and moments of inertia. Each of these is regarded primarily as a limit of a sum.

The Appendix (pp. 179-204) contains formal proofs of certain propositions which in the text are made evident by rather informal discussion.

What is needed of trigonometry and analytic geometry is introduced as required. It is rather surprising to find that the 178 pages of the body of the text could surely not have been shortened by more than 25 pages if full knowledge of these subjects had been assumed.

For years past Klein in Germany, Perry in England, Moore in the United States, and hosts of others have been urging upon the mathematical fraternity the possibility and desirability of introducing the notions of the calculus at an earlier stage than is now done. In view of this, the little book under review is particularly significant. It gives a concrete means of judging whether such simplifications as would be demanded by distinctly less mature classes would be possible. The reviewer dare not undertake to say that this is an ideal book for very young classes, or even as well fitted to the American situation as we might reasonably expect were the book written expressly for us; but barring a few minor features closely related to English usage where it differs from ours, and other features which are related to the particular class of students for which it was primarily developed, it seems that the grade of difficulty is adapted to the fourth year of our secondary schools or the first year in college.

Would not a one year course of college mathematics be vastly more interesting and instructive if based upon a book of this type than the present combination of trigonometry, college algebra, and analytic geometry? In a majority of our secondary schools we now have only three or at most three and a half years of mathematics. Would not such a course as this furnish admirable material with which to complete a full four years course?

Unfortunately the typography of the book is ineffective and the pages are monotonous in appearance. The page architecture is particularly bad. An important figure frequently occurs near the bottom of a right hand page with nearly all the discussion on the next page. But such minor defects aside, one may reasonably hope that this little volume points the way to a vitalizing of an important part of our mathematical curriculum.